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USAAVLABS TECHNICAL REPORT 70-21

SOME EXPERIMENTAL STUDIES ON THE RELATIONSHIP BETWEEN END FIXITY AND CRITICAL LOAD LEVEL FOR STRUTS

By

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August 1970

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

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STANFORD UNIVERSITY
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This program was carried out under Contract DAAJ02-68-C-0035 with Stanford University under subcontract to Georgia Institute of Technology.

The data contained in this report are the result of research conducted to verify the analytical evidence that the end fixity for a compressed column can be determined from a lateral load test of the same structure. Experimental tests were conducted on a series of simple, rectangular, cross-section struts with various combinations of fixity at the two ends of the column.

The report has been reviewed by the U.S. Army Aviation Materiel Laboratories and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

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SUMMARY

Experimental evidence is presented which shows that the critical load for a compressed column can be approximated closely for a wide range of rotational end fixity conditions using a simple lateral load test of the structure. A series of tests was conducted on each of two columns over a range of rotational end fixity. Axial load versus lateral deflection data was plotted according to the Southwell method for each end fixity condition. The results of these tests were compared to lateral load/lateral deflection test results for the same end fixity conditions.

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LIST OF SYMBOLS

P	axial compressive force, lb
P_{cr}	classic critical load for column, lb
E	Young's modulus, lb /in.^{2}
I	minimum x-section modulus, in.^{4}
L	column length, in.
K	end fixity coefficient
f	flexibility coefficient, in./lb
W	concentrated side force on column-beam, lb
δ	midpoint deflection of column under axial compressive force P, in.
Δ	lateral deflection at point of application of W, in.

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INTRODUCTION

The subject of the influence of boundary restraint on the stability of structures is one which has always been of concern to engineers.

When a relatively slender, geometrically perfect, centrally compressed column is fixed at both ends by a device which does not permit lateral motion of either end, the critical load level is given by the classic Euler formula

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Increasing the degree of end fixity by imposing, for example, a rotational restraint at either end increases the critical value. The maximum load-carrying capability results when the end rotational restraint is such that the curve of lateral displacement under axial thrust has zero slope at each extremity of the column. In this case, the load value is four times that of the simply supported structure. If constraint is relaxed from the simply supported condition at one end, even though constraint may be increased at the other, the crippling load is reduced. The instability load level, for example, for a column encastre at one end and free at the other is a mere 25% of the value for a simply supported column.

These wide variations in load-carrying capabilities that result from changes in end conditions, have caused experimentalists over the last century to devote much time and effort to devising end restraint fixtures which would give a close approximation to the idealized boundaries referred to above. This work started with the earliest tests of Hodgkinson[1] and has continued unabated since that time. A recent paper[2] reviewed the many studies made and concluded that it would be more profitable to concentrate on methods for nondestructively evaluating actual end fixities rather than on attempting to further refine the test procedures. This finding was further enhanced by the knowledge that, in practice, the engineer has little to guide him in relation to the prediction of the degree of imperfection common in practical direction-fixed ends. In reality, there has been little change in this regard since Salmon[3] commented in his work on columns in 1921.

The most pressing point for future research on the subject of columns is undoubtedly the question of the degree of imperfection common in practical direction-fixed ends; in short, what value of K ($P_{cr} = K.\pi^2 EI/L^2$) should be assumed for such ends? A complete answer to this question is difficult, but at present the designer has no real data whatsoever regarding practical end conditions.

To meet this pressing need, Horton, Craig, and Struble undertook a series of analyses[4] and showed that, for a wide range of end fixities, a simple method for evaluation of the fixity coefficient seemed feasible. The result that they obtained was that the product of the critical compressive load and the maximum flexibility coefficient for a point lateral load, applied to the same structure, should be, to all intents and purposes, dependent only upon the length of the column. The value of the "constant" so defined is $\pi^2 L/48$, approximately.

The prime purpose of the work reported here was to determine whether the analytical results were borne out in practice. To this end, a series of simple rectangular cross-section struts was tested with various combinations of fixity at the two ends of the column.

EXPERIMENTAL SETUP

The test columns used in this investigation were fabricated from either 2024 aluminum alloy or mild steel rectangular cross-section bar stock. The cross-section of the aluminum column was 1/2 in. by 3/4 in. and that for the steel was 1/2 in. by 5/8 in. Knife edges were milled on the ends of the columns to approximate simple support conditions. Then, rigid cross-members were attached to the columns 0.25 in. from either end to form a base through which to apply end restraining moments. These end moments were generated through reaction on steel cantilever springs of various stiffnesses. The system is illustrated in Figure 1. It bears a great similarity to that adopted by Osgood[5] in his classic study of 1938 (see Figure 2). The prime difference between the systems is that Osgood used compression springs instead of leaf springs.

The test specimens were mounted in a 20,000-lb-capacity Instron Universal Test Machine with the knife edges laterally restrained by hardened steel V-blocks. Lateral forces on the column were applied through and transduced by a load cell. Deflections were measured with a sensitive dial gage. Output from the load cell was processed by a digital voltmeter scaled to indicate directly, in pounds, the force applied. The load cell and the dial guage were supported in such a fashion that their vertical positions could readily be adjusted. The whole system is illustrated in Figure 3.

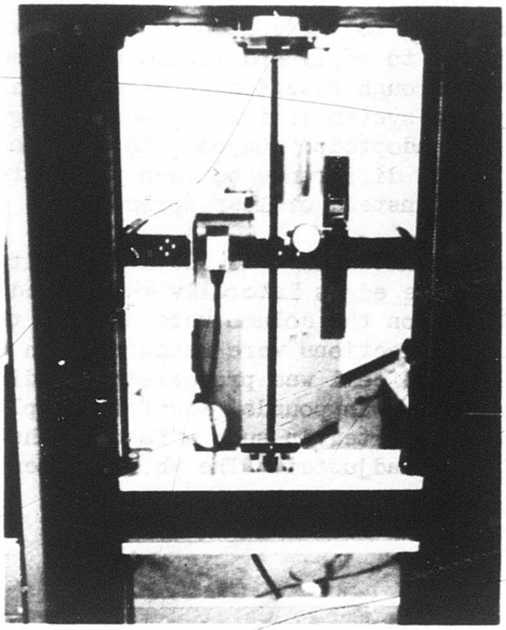

Figure 1. Experimental Column Mounted in Test Machine.

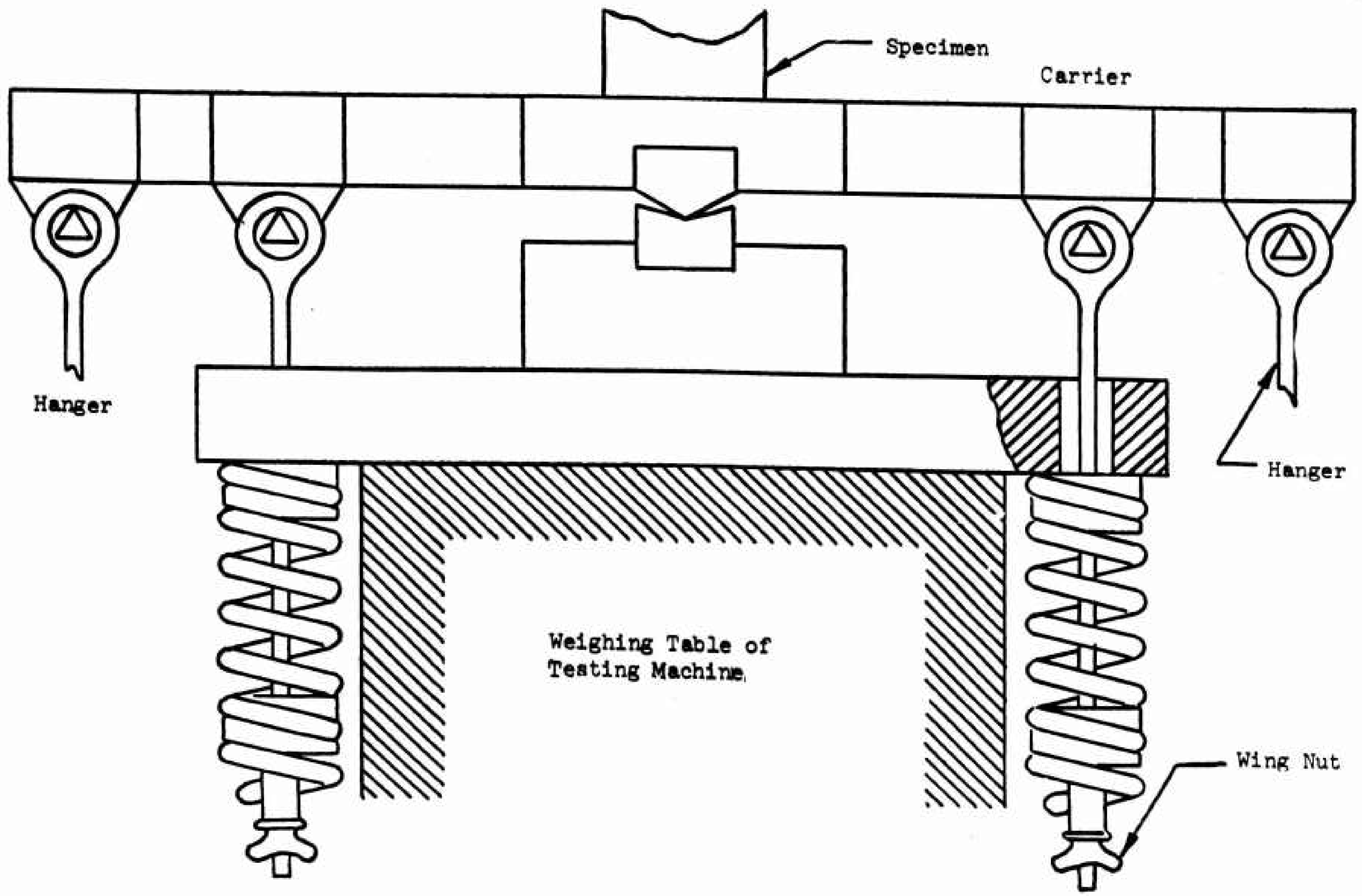

Figure 2. Experimental Test Column (Osgood, 1938).

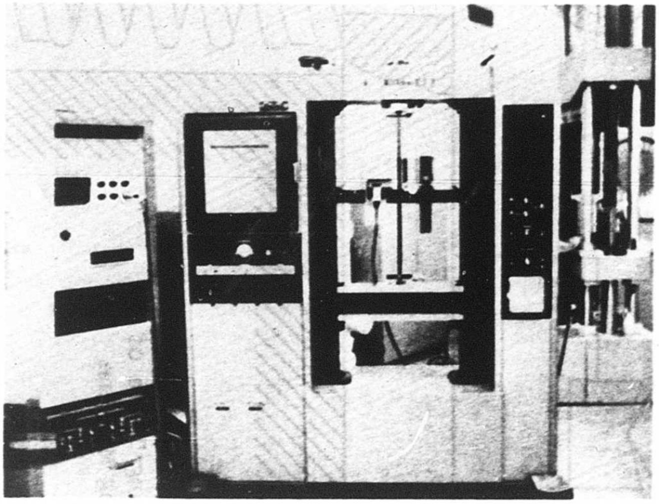

Figure 3. Test System.

TEST PROCEDURE

On each column, tests were performed for a wide range of end restraint conditions. For a particular specimen, the variations in end fixity were made without removing the column from the machine. Thus, deviations due to change in alignment of the knife edges, etc., were minimized. Relative end fixity for each test and identification of test numbers are given in Table II.

Values of lateral force/lateral displacement corresponding to zero axial load were obtained for each end restraint condition at a sufficient number of positions along the column to locate and determine the maximum flexibility coefficient for a lateral concentrated load. Then, values of lateral deflection versus axial load were determined for the same setup.

A test series started with the column restrained only by the knife edges at each end. Then, without altering the conditions at the upper end, the lower rotational restraint was increased. Three successively higher values of restraint were used here. Next, with the lower restraint held at the maximum level attained in the previous tests, three levels of rotational constraint at the upper extremity were investigated. A final test was made on the aluminum column with the upper restraint removed and the lower end essentially clamped.

TABLE I. RESULTS OF COLUMN TESTS			
ALUMINUM COLUMN			
Test	P_{cr}, lb	$(10^2)f$, in.	$P_{cr}f$
1	942	.675	6.35
2	1160	.550	6.37
3	1272	.520	6.60
4	1498	.434	6.48
5	1795	.367	6.58
6	1990	.290	6.62
7	2240	.332	6.50
8	1750	.384	6.72
STEEL COLUMN			
Test	P_{cr}	$(10^2)f$	$P_{cr}f$
1	1630	.463	7.55
2	1867	.406	7.57
3	1987	.374	7.44
4	2299	.324	7.44
5	2577	.290	7.54
6	2771	.274	7.58

TABLE II. END RESTRAINT CONDITIONS FOR TEST COLUMNS

ALUMINUM COLUMN

Test No.	End Restraint Conditions
1	Both ends, simple support
2	Upper end, simple support, lower spring, step 1
3	Upper end, simple support, lower spring, step 2
4	Upper end, simple support, lower spring, step 3
5	Upper spring, step 1, lower spring, step 3
6	Upper spring, step 2, lower spring,step 3
7	Upper spring, step 3, lower spring, step 3
8	Upper end clamped, lower end simple support

STEEL COLUMN*

Test No.	End Restraint Conditions
1	Both ends, simple support
2	Upper end, simple support,lower spring, step 1
3	Upper end, simple support, lower spring,step 2
4	Upper end, simple support, lower spring, step 3
5	Upper spring, step 1, lower spring, step 3
6	Upper spring, step 2, lower spring, step 3

*Southwell loads for end restraint conditions corresponding to Tests 7 and 8 for the Aluminum Column could not be obtained for this specimen with the instrumentation available for these tests due to the narrow range of loading over which the prebuckle deformations developed.

DATA PROCESSING

The critical loads for struts were obtained from Southwell[6]. In this procedure, the ratio δ/P is plotted versus δ , p being the axial compressive load and δ the corresponding elastic deflection at the midpoint of the strut. P_{cr}, appropriate to the achieved end condition but connected for other imperfections, is then computed from the slope of the δ/P versus δ line. The results of this portion of the study are shown in Figures 4 and 5.

The maximum flexibility coefficients were determined by plotting lateral load W versus deflection Δ for the spanwise location which produced the maximum slope for such a plot. This portion of the data is shown in Figures 6 and 7.

Values of the critical load (P_{cr}) multiplied by the maximum flexibility coefficient, f, for each strut and every end condition investigated are shown in Table 1. Percentage variation of the quantity $P_{cr}f$ from the quantity $\pi^2 l/48$ is shown in the right hand columnof Table I.

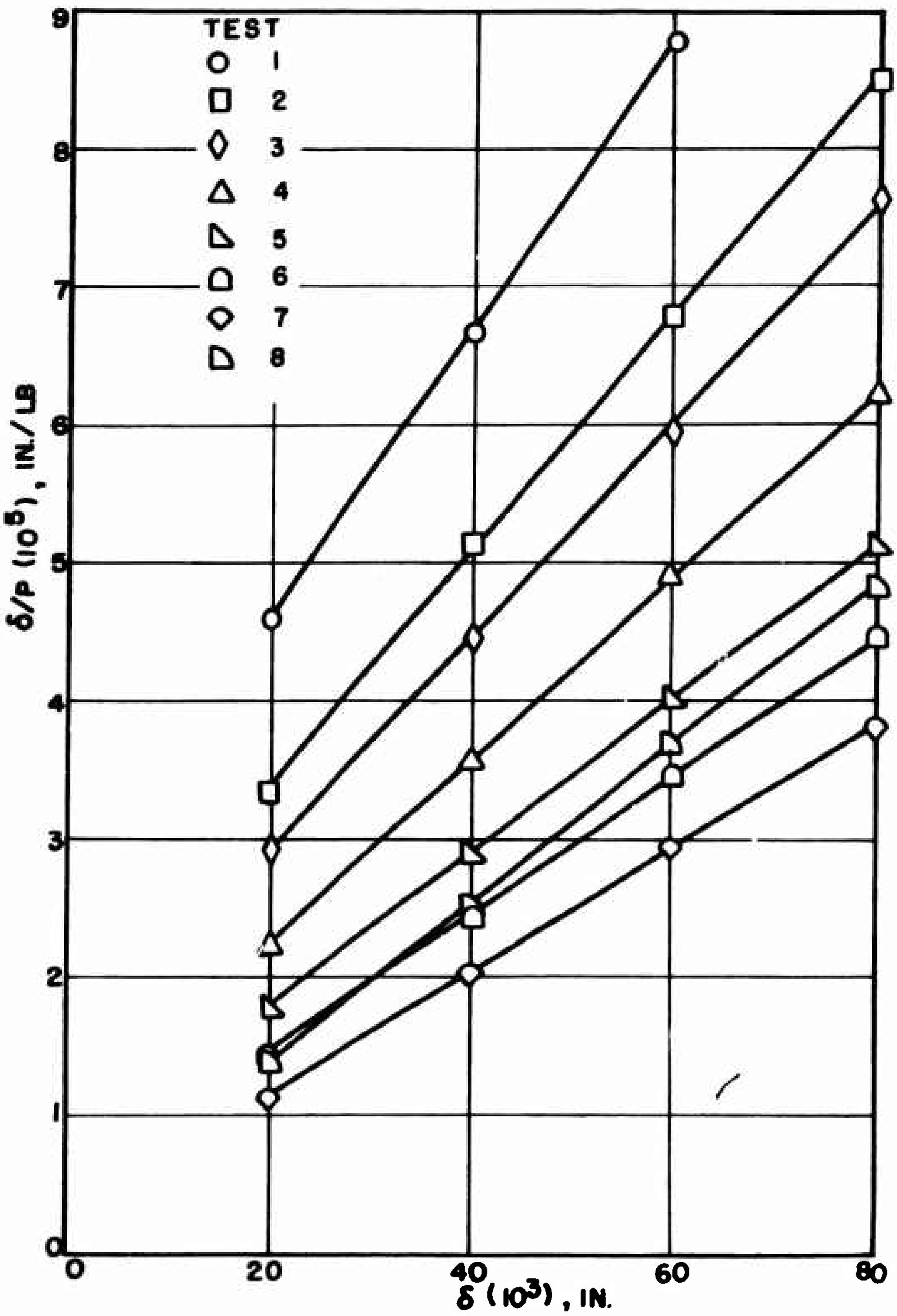

Figure 4. Southwell Plot - Aluminum Column.

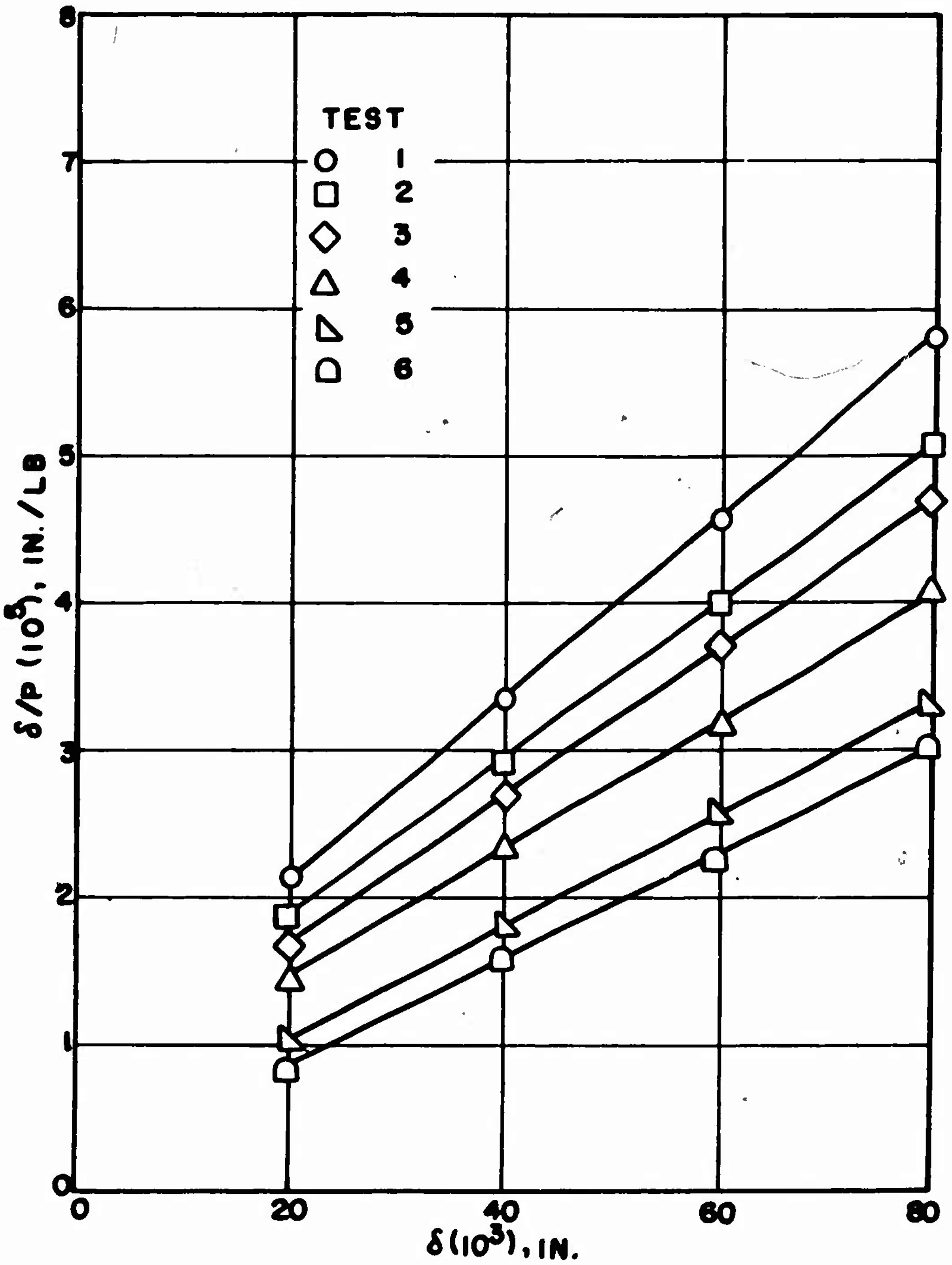

Figure 5. Southwell Plot - Steel Column.

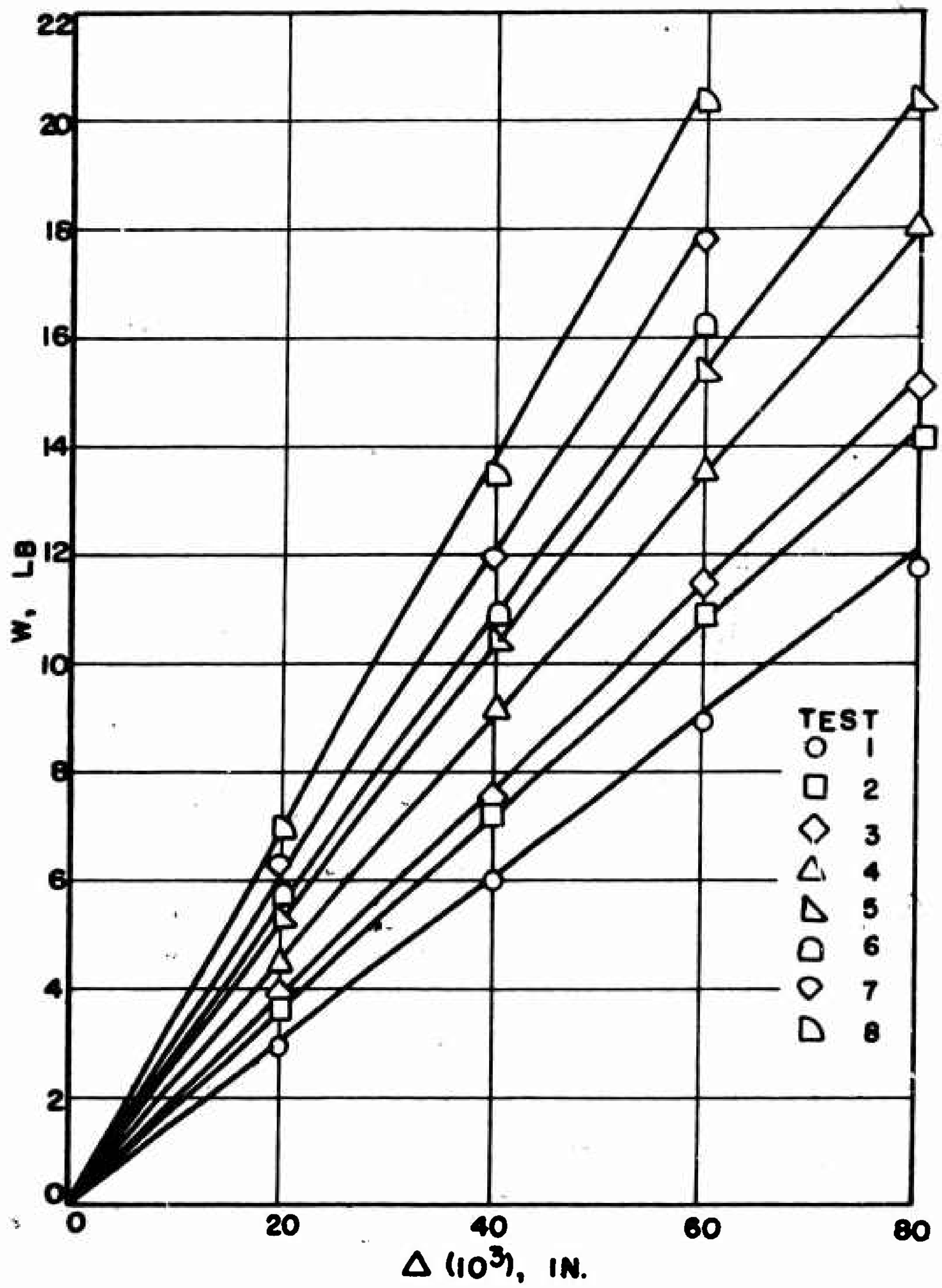

Figure 6. Lateral Load/Deflection Curves - Aluminum Column.

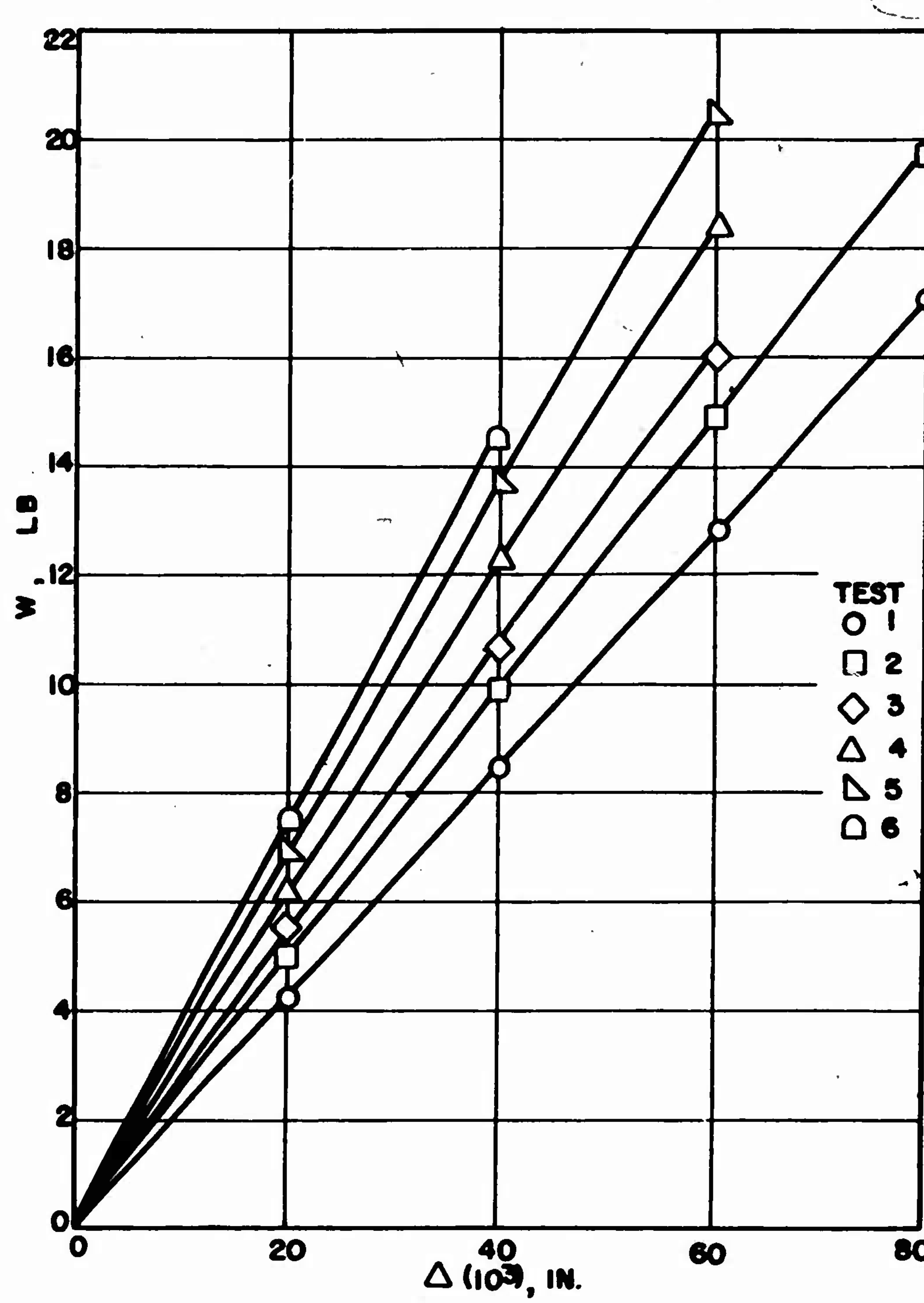

Figure 7. Lateral Load/Deflection Curves - Steel Column.

CONCLUSIONS

The tests reported indicate clearly that it is both feasible and simple to approximate the critical load for a column over a wide range of rotational end fixity conditions by determining the maximum flexibility coefficient for a concentrated lateral side force and dividing this quantity into a characteristic number for the column. As the results given in Table I show, the product $(P_{cr}f)$ is substantially constant for a particular column irrespective of end conditions.

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13. ABSTRACT

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